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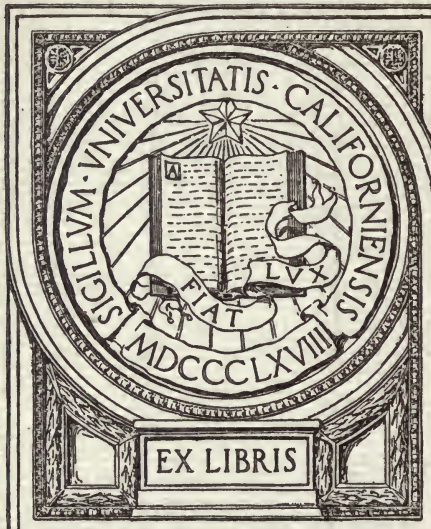

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MECHANICAL ERRORS

—IN—

The Common Theory of Flexure

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MECHANICAL ERRORS

—IN—

The Common Theory of Flexure

—BY—

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General Principles Applicable to Balanced Parallel Forces.

Before discussing the mechanics involved in the "common theory of flexure," it is necessary to restate some elementary principles relating to the action of forces, and to define certain distinctions in technical terms, drawn from practical mechanics, that are not found in text-books of the subject.

1st. In a balanced system of parallel forces the sums of the forces acting in opposite directions must be zero; in other words, the sum of the vertical forces must be zero and the sum of the horizontal forces must be zero.

2nd. The algebraic sum of the moments relative to any axis shall be nothing.

3d. When two equal and opposite forces balance, whose lines of action do not coincide, a shearing force must be transmitted from the line of action of one to that of the other, this *shearing* is the right line balancing agent.

4th. The lever-arm of a force is the distance traveled by its shearing force in order to reach a centre of motion or tendency.

5th. A shearing force cannot be transmitted by a body through and beyond a center of rotation or tendency.

6th. A shearing force has no leverage and therefore no moment.

7th. A force to possess leverage must transmit its shearing force to a center of rotation or *tendency*, where it must be opposed by an equal and opposite force.

8th. A body, and therefore any part of the body, is at

rest in any direction, when each particle of one of its planes is solicited to move in opposite directions by pairs of equal and opposed *conspiring* forces, parallel to the given direction.

9th. Two equal and opposed shearing forces will produce compression in, and if sufficiently intense, *crush* the body that transmits them.

10th. When an unbalanced force is applied to a beam and equilibrium is established by the development of internal forces, the originally unbalanced force must be equilibrated by its own generated components, and not by those of some other applied force.

The 1st and 2nd conditions are those usually given as *solely sufficient* to determine the conditions of equilibrium of any body, the others are none the less universally true, and are here formulated for the convenience of reference, as their existence is very frequently ignored by writers, in their discussion of mechanical problems.

Moments.—The moment of a force about a point measures the *tendency* of the force to produce rotation about the point, when one point of the body is "fixed" the *tendency* to rotation is around this point, which is therefore the origin of moments.

Theoretical mechanics takes the moment of a force with respect to any point, whether there is a *tendency* to rotate around it or not, in the first case the moments possess leverage or mechanical power, in the second they do not; theoretically, the moment of a force may have an infinite number of values, but in practical mechanics it has value only when taken with reference to a center of rotation or *tendency* in the body to which it is applied.

Theoretical moments, from an "anywhere origin," are sufficient to determine the conditions under which a *whole* body may be *at rest*, relatively to the other bodies of the world; but when we equate *power* and *resistance*, as in a solid beam, the moments must possess mechanical power, such as Archimides would have used had he turned the world over, which power they only have when taken with

reference to a *common center*, around which they all *tend* to rotate, and around which the body *could rotate* if the forces that established the condition of *tendency* were removed. The location of this center clearly classes our forces into *power* and resistance, the failure to do this in the "common theory" causes the *tension* to be the *resistance*, and the *power* to be the *load* and the *compression*, as their moments add, having like signs, which of itself condemns the theory.

In nature, the moments of two applied forces, whose signs are unlike, never add, as the mechanical power expressed by each moment tends to do opposite or unlike things. The only exception to this invariable rule is on paper, where the moments of the unbalanced forces of a static couple are said to add; but the moments of these forces possess no mechanical power, only a possibility of such, which is only realized when the forces balance as right line forces, and then one force has no moment as its lever-arm is zero. The application of this mythical mechanical moment of the unbalanced forces of a couple to the tension and compression in a solid beam has lead to a palpable violation of the fundamental principle above stated. The moment given a static couple violates the condition of *tendency* which is the very essence of moments.

Shearing Force and Leverage.—Shearing is a peculiar auxiliary force that is found to exist when an applied force has leverage; it travels from the point of application of the force to the center of rotation or *tendency* to such rotation, it enables the force to possess leverage and to balance another opposed equal force when their lines of action do not coincide.

At any section between the point of application and the center of rotation, it is equal in intensity, parallel to the force, and has the same direction—its tendency is to cross-cut the body that transmits it at each sectional plane.

A shearing force cannot possess leverage, its existence being the result of leverage, it cannot in turn reproduce its own cause.

The forces that have a shearing force are *purely static*, such as a single force that tends to produce rotation, and a pair of opposed tensile forces that tend to resist rotation, if these forces have no shearing they neither *tend* to produce nor to resist rotation; for they then have no leverage. But neither of the two opposed compressive forces have any shearing force, and the leverage they possess is direct without its assistance. A compressive force has this leverage in resisting the rotating tendency of another force, only when their *tendencies* are in opposite directions around an actual (not theoretical) center of tendency to rotation. In Fig. 1, if the actual center of rotation is below the point of application of the compressive force C, as it really is, then the force C has no leverage in resisting the rotating tendency of the load L.

Observation teaches that a body cannot transmit a shearing force a greater distance, than that, from the line of action of the force to the center of rotation, and that this distance traveled by the shearing is the limit to the length of the lever-arm of the force.

The beam A, Fig. 1 presses the support S through the shearing force L, and the support presses the beam with an equal and opposite force, as this force S may also be the shear of another force, these two equal and opposed shears will produce pressure equal to that had the two forces been directly appiled at S. A familiar illustration of this is seen in the ordinary nut-cracker, the pressure on the nut is the sum of the shears of the force applied by the hand and the force developed at the rivet of the hinged joint.

Couples.—Two equal and opposite *unbalenced* forces, whose lines of action do not coincide constitute a static couple, and the body to which it is applied should be in equilibrium, as the 1st condition is rigidly fufilled, but the effect of a couple is to rotate the body about its center of mass, hence it is not in equilibrium. However, the instant the body, from any cause, is enabled to transmit a shearing force, or fulfill our 3d condition, equilibrium is es-

tablished, but the pair of forces cease to form a couple and become a part of a common lever where one force is the *power*, the other the *fulcrum* and the *resistance* whatever enabled the body to transmit a shearing force.

A "static couple" is a misnomer, the rotation produced by it about the "center of mass" is dynamic, which cannot be converted into a *tendency* about the same "center of mass" for the instant it becomes static equilibrium the center of tendency to rotation shifts from the center of mass to the point of application of one of the forces that previously formed the "static couple." The application of the "parellelogram of forces" to the equilibrated forces will demonstrate the truth of this, or it fulfills our 7th condition.

Theoretical mechanics considers any two equal and opposite forces to form a couple that either rotates or *tends* to rotate the body about its center of *mass*, whether the forces are balanced as right line forces or not, This is evidently erronious, for if the forces are balanced it is physically impossible for the body to transmit the shearing force (the right line balancing agent) *through* a center of rotation or *tendency* to such rotation, it can transmit it to such a center, but not through it, hence if the forces tend to turn the body about its center of mass they are *unbalanced.* This is an especially important distinction to make, since a writer, who is usually very clear, states that the forces (L, S) Fig. 1 form a couple that tends to rotate the body A, about its center of mass, thus making the shearing force L travel through a center of tendency to rotation, and a point that is "fixed" by the opposite forces, S, and shearing force L, *tend* to travel in the arc af a circle around this center, both of which being a mechanical impossibility. The rule of statics so often quoted by authors in the analysis of the common theory, that "if two couples applied to the same body balance each other they must have opposite algebraic signs," and its corallary, if the forces in the direction of one axis reduce to a couple those in the direction of the other must reduce to a couple also, for equilibrium has led them

to erronious conclusions. The rule is only true when the pairs of forces balance as right line forces, and thus cease to be static couples, and the resultant moment of each pair is the leverage moment of a single force and its corallary, when the resultant moment of one pair reduces to the moment of a single force, the resultant moment of the other pair must reduce to the leverage moment of a single force with opposite sign.

To illustrate (L, S) and (T C) Fig. 1 are said to form such couples. Opposed to S there is a shearing force L, equal to it, our couple (L, S) in balancing the forces has the vertical effect of three forces, the resultant moment of which, according to a rule of statics, is the moment of the single force L. In the same way, horizonally, we have the effect of three forces, T and C and shearing force T, equal and opposed to C, the resultant moment of the three is the moment of the single force T.

When the resultant moment in the direction of one axis does not reduce to that of a single force, as the tension and compression in a beam, the mechanical necesssity for and their ability to develop equally does not exist. In this case the line of direction of neither force passes throngh the centen of rotation. The line of action of the compressive force C being *between* the line of action of its companion force T and the center of rotation, the point of support, its moment and that of the rotating force must have opposite signs, and T must have the same sign and direction of C, if it is an "applied" force.

THE COMMON THEORY OF FLEXURE.

It is a misnomer to call the common theory "a theory of flexure" for the theory itself does not recognize the effect that is produced upon the interior of the beam by bending in the slightest degree. It is true from knowledge obtained outside of the teaching of the theory, the total amount of the interior forces is derived from considering them to vary uniformly from a neutral line, but this is also done

when the beam is not bent at all, then it can no more be called a "a theory of flexure" than it is of any other structure in which the stresses are equal, such as bridge trusses, etc.

To be a theory of flexure it must recognize that the stresses before flexure begins, exist in a different condition from that after, and that flexure caused them to change this condition and to assume the order of arrangement that is known to exist in a bent beam.

In its mechanics the common theory is based up undisputed principles of *pure* statics, but when it is attempted to explain the phenominon known to attend flexure it can only be done by adopting, at every point, principles just the reverse of its teachings, and it is a mechanical error to attribute to it effects that it cannot produce.

In reasoning from an effect to its cause the human mind more readily accepts false logic as true than when it adopts the reverse process. The best authorities emphasize the statement "that a body in static equilibrium is conceived to be incaple of *bending*, *breaking* or *changing* its shape in any way, and their effect is not considered," yet from the knowledge of what takes place in "bending," "those who ought to know how to apply principles" to effect has described its cause as being "a clear case of statics" when the reverse process of reasoning would have denied the effect claimed at every point. To the extent that a beam rotates or bends under a transverse or a column deflects under a longitudinal load, it is *actual constrained rotation*, this induces actual longitudinal molecular movement, which, of itself, takes it out of the jurisdiction of static laws. Static equilibrium is only a *tendency* to rotation, which rotation, if it took place, would be free without molecular movement.

Static forces acting in a single plane are *always* equilibrated on the principle of the *parallelogram of forces*, when their resultants intersect. In the parallelogram *only* two forces possess mechanical power, and the equality of their

moments prevents rotation, but the common theory conceives that three forces have this power ; as the compression force simply prevents *horizontal* translation, it may be left out, just as the companion force that prevents vertical translation, as neither have any lever-arm.

In the "parallelogram of forces" the center of tendency to rotation is at the end of the diagonal where the forces are balanced by their *reactions*, which in the solid beam is a point on the resultant of the compressive force, but there can be no *rotation* around this point and therefore no *tendency* as the two are inseparable, which destroys the commom theory equilibrium, as it must be around some point outside of the parellelogram.

Generated forces that produce compressive *strain* and possess mechanical power or leverage are not recognized in statics, and its treaties make no attempt to trace the mechanical relation that must exist between the generating and generated forces, therefore the rules that formulate the principles of statics do not apply when this is the case, or more than two intersecting forces possess mechanical power in an equilibrated system.

The "common theory of flexure" is deduced from the following conditions. Let a beam be loaded at its ends and supported at the center, then after equilibrium has been established conceive one-half of the beam to be removed, and the balanced condition destroyed. Then it is conceived that we have, only, an unbalanced force L, and a rigid body A, and that we must by external applied forces re-establish, *de novo*, the equilibrium that existed between the external force L and the developed internal forces, which we destroyed in the cutting process.

This gives the diagram of forces represented in Fig. 1. From the 1st and 2nd conditions given, we have L=S, and T=C, and the moment of the (L=S), system equal to the moment of the (T=C) system.

That T must equal C as the *sole* and *sufficient* cause of equilibrium is only true when the forces are concurrent and

their lines of action coincide, but here a distance intervenes between them, and in order to balance they must get together in some manner, and be opposed just as if they were concurring and their lines of action coincided,—this the beam A enables them to do by transmitting a *lōngitudinal* shearing force from T to C.

To follow the line of argument of a writer[1] on this subject—conceive that we have only the forces L and T, vertically, the 1st condition must be fulfilled, viz: "L=S since the pressure upon the fulcrum must equal the dormant force L" or its shearing force. "To prevent rotation we

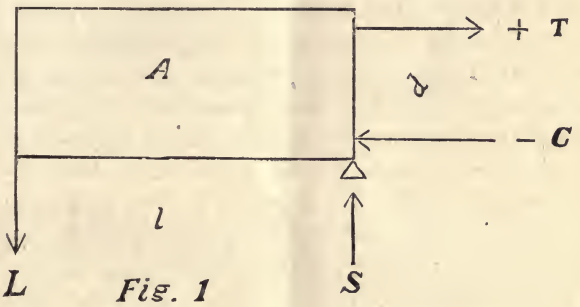

Fig. 1

must now have Ll=Td. But these two conditions are not sufficient unless we introduce a new force C from outside and make C=T, we shall have horizontal motion." From his failure to state how T balanced C, the writer here loses the thread of his argument and thus reaches a conclusion entirely at variance with the premises laid down by himself. He should have said, to be consistent, "the pressure of the half beam A against" and opposite the "new force" C must be equal to the "dormant" or shearing force T. And as shearing force T, that is equal and opposed to C, is not a "new force" but the same force T in a new place, he would have reached the conclusion that the load L has only one horizontal component instead of two, and the moment

(1) Prof. DuBois, Eng. News, Jan. 21st, 1888.

of resistance is the moment of a single stress instead of that of a stress couple."

The writer also erred in giving the 1st condition as his reason for making T—C to prevent horizontal "traveling", in the whole beam, that is accomplished from the forces complying with our 8th condition, which is *solely sufficient*, and it was a fatal error to "conceive" that the part could "travel" independently of the whole. His true reason for this is found in our 7th condition, in order that T may possess leverage to resist rotation.

The advocates of the "common theory of flexure" insist that the tension must balance the compression in the beam, just as the force at the supports balances the load, and as their text-books on mechanics demonstrates no other method of balancing except through the aid of an auxiliary shearing force, then they must accept all of the absurdities and contradictions of this method when applied to a solid beam.

Distribution of the compression.—It is rightly claimed by "the common theory" that at any section the *tension* is distributed through the portion of the beam that it occupies as an uniformly varying force. When the resultant T balances C the laws of mechanics require that the shearing force T shall be greatest at *n*, Fig. 2, where the common theory requires its opposing force to be zero. The vertical shearing force is a maximum at the inner side of the supports, this horizontal shearing force must follow the same law, and be uniformly distributed along the line *ns* instead of as an uniformly varying force. Wiesbach states that *n* is the center of rotation—this limits the lever-arm of T to its distance from *n*, and this will be all of the moment that sustains the load, as it is force T that posseses leverage and not shearing or "dormant" force T.

The neutral line destroyed.—The method of balancing T and C, adopted by "the common theory" requires that two equal and opposite shearing forces T (or C's after they became opposed) shall pass through the neutral line, the ef-

fect of which will be to destroy the properties claimed for the line by the theory.

Is it not a mechanical absurdity to say that T and C balance each other horizontally as right line forces, when their lines of action do not coincide and that their is between their lines of action a *neutral* or *dead line* over which the effect of neither force can pass ? The equal but unbalanced forces of a "static couple" that simply overcomes

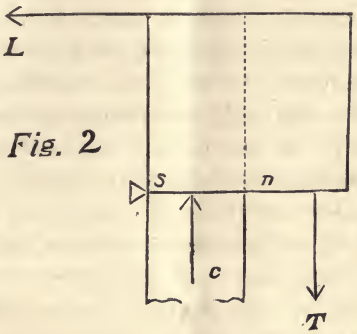

Fig. 2

the inertia of the body and causes it to rotate about its center of mass has what resembles the neutral line, but it disappears as soon as the forces balance each other by the bodies transmitting a shearing force.

A solid beam has a neutral line and the compression varies uniformly, neither of which could exist, as such in the beam, at the same time with a longitudinal shearing force. The tension cannot balance the compression without this force, therefore they do not balance, and the theory must be reversed in order to explain the facts.

Method of Sections, or of Substituting "applied" for generated Forces.— The Railroad Gazette[2] says of my objection to the establishment of the "common theory" by what is known as the "method of sections" that "this mode of procedure is so well understood in the treatment of stress that

(2) September 6th, 1889.

it seems unnecessary to refer to it unless we have made an error in its application," this error I think is evident.

"The applied forces and horizontal stresses on each side of any section of a loaded beam hold each other in equilibrium" *each on its own side of this section.* Now if we conceive one part of the loaded beam to be removed, at any section, and the existing equilibrium in the part that remains to be destroyed, or it is reduced to a *rigid* body and an unbalanced load, then the horizontal forces that we must "apply" at this section to establish horizontal (right line) and rotary equilibrium, at the same time gives us "a clear case of statics." This is the "well recognized" condition of equilibrium when the vertical applied force generates nothing but free rotation or its *tendency*, and it certainly cannot be identical with its condition when it generates constrained rotation accompanied by horizontal stress through "deformation," therefore the "common theory" leaves out of consideration, entirely, the effect of the generation upon the generating cause when it substitutes the right handed applied couple (T C), Fig. 1 for the left handed generated couple in A, for as mechanical powers they are of entirely reverse action.

But if we conceive, as we must, that the same equilibrium that existed in the half beam *before* removal must exist in it, *after* removal, as we have taken away neither the cause, the "applied" load, nor the effect, the generated stress, on its own side of the section, which equilibrated each other in the whole beam, and therefore must do so in the half beam, then it is not such "a clear case of statics." In each half of the beam, after being divided, we will have a vertical applied force, whose moment has been exhausted or equilibrated by the moment of its two generated horizontal components, otherwise our cutting and removing will introduce an element that did not exist in the loaded beam as a whole, viz: the unbalanced moment of the vertical load, which we are not warranted in doing for the pleasure of balancing if again in some other manner.

The *bending* of a beam, under a tranverse load, is *actual rotation* around some point, and not simply a *tendency* around every known point, were it not so the stresses could not arrange themselves on each side of the neutral line as as they are known to do. Statics does not recognize that there can be any motion in an equilibrated system of forces by reason of the "deformation" of the body upon which it acts, but conceives that the body cannot *bend* or *break* under the action of any force, however, intense, but always retains its size and shape unchanged.

If the rotation of the beam A, Fig. 1 by the load L, generated nothing but free rotation or *tendency*, which is all that is contemplated by statics, then "the common theory" method of balancing by the section method is correct. But the rotation or *tendency* instead of being *free*, where every point in the body *tends* to move in the arc of a circle with a radius equal to its distance from the common center, it is constrained rotation where every point actually moves in a curved line, with a constantly changing distance from "the common center," from each such point having a longitudinal motion.

In free rotation or its tendency the load generates no stress components as there is no horizontal motion, and its tendency to freely rotate the beam in one direction must be resisted by outside "applied" forces that tend to rotate it, equally, in the opposite direction, as in the common theory, but the constrained rotation of the beam generates two horizontal stress components, and the effort expended by the load in their generation equilibrates and absorbs its rotating power, hence there is no unbalanced *rotating tendency* for our outside "applied" forces, at the section, to balance by *rotating* the beam in the opposite direction.

The method of sections will lead to no error if it is simply used to *interpret* the equilibrium that is established through the generation of horizontal forces instead of establishing it *de novo* as is done in the "common theory." In this interpretation two separate and distinct uses must

be made of the "applied" forces,—1st, to apply such forces that will establish the horizontal equilibrium that existed in the whole beam, 2nd, to apply such forces that their mechanical power, or moment, will resist the rotating power of the load in the same identical way that it was equilibrated by the moments of the generated stresses on its own side of the section of division, and both objects cannot be accomplished at the same time by the same diagram of forces as is done in the "common theory."

The first problem then is to apply at the removed section two such force systems, of which T and C are the resultant Fig. 1, that will enable the half beam A to sustain the load L, with the same generated stresses in A that existed before the removal of the other half of the beam. After division the horizontal stresses must be conceived to exist *in each* part, just as they did before division, and as it would be impossible for these stresses to import motion to the *half* beam in which they exist, whether they are equal or not, there is no mechanical necessity for our making our applied forces equal. We must then find only such forces that must have existed in the removed portion of the beam in order that such forces when applied will produce with the forces already in the half beam, the observed horizontal deformation, that is, the elongation by tension and the shortening by compression and the original horizontal equilibrium will be restored by applying a *pull* equal and opposite to the existing *pull*, and a push equal and opposite to the existing *push*, and this is the only condition we can legitimately impose upon these forces, when they are equal opposed pairs and conspiring at any section. "The applied" forces T and C, Fig. 1 represent in the system the removed load on B, and thus enables A to sustain the same load that it did before removal and with the same generated stresses in A.

The second problem then is to find two such forces that when *substituted* for the left handed couple generated in A, by the load L, Fig. 1, the mechanical action of the substi-

tuted or "applied" and the generated forces will be *identical* in resisting the rotation of A, produced by L. All of our knowledge of mechanical power is derived from experience and not from a priori reasoning, and we must follow the same teacher when we substitute the moment of an "applied" force for the moment of a "generated" force. A generated *tensile* force and the generating force have the same sign, and if both were applied or possessed rotating power they would rotate the beam in the same direction, but a generated *compressive* force and the generating force have opposite signs and tend to rotate the beam in opposite directions. The rule for the *substitution* then is, *the moment of a generated tensile force is the moment of an "applied" force in the opposite direction, and the moment of a generated compressive force is the moment of an "applied" force in the same direction.* The "common theory" recognizes the correctness of this rule when it substitutes the moment of the "applied" force T, Fig 1, for the moment of the opposite generated *tensile* force in A, but ignores it, and thus the teachings of experience when it substitutes the moment of the "applied" force C, for the moment of the opposite generated *compressive* force in A, and thus arrives at an erroneous conclusion. For resisting the rotation of A, it requires a force C in the same direction with T.

Equilibrium at the Section of Maximum Stress.—Conceive the beam to be divided, at the section of greatest horizontal stress, and the forces to be balanced in each part as claimed by the common theory, then place them together but separated by the rubber body as shown in Fig. 3. The lines of action of the (T T) and (C C) forces are represented as not conciding so as not to obscure the action that is claimed for them.

Now if + T balances — C, it can only do so from the body A, transmitting the auxiliary shearing force of + T, to the line of action of — C, and this becomes + C, which is the shearing of + T, and if — T balances + C, it can only do so from the beam B, transmitting the shearing force

ot — T, to the line of action of + C, and this becomes — C. Then the two forces + C and — C are the shearing forces of the pair ot T forces and compress the rubber body just as if they had been originally applied to it.

The common theory makes — T sustain one-half of the load on B and its shearing force — C, one-half of that on A. Now force — T, and its shearing — C, are in nature and mechanics the same identical force, this theory then makes force — T exhaust it power to sustain the load twice over, at the same time, or in other words, it sustains the whole load on either A or B, which is true in this method

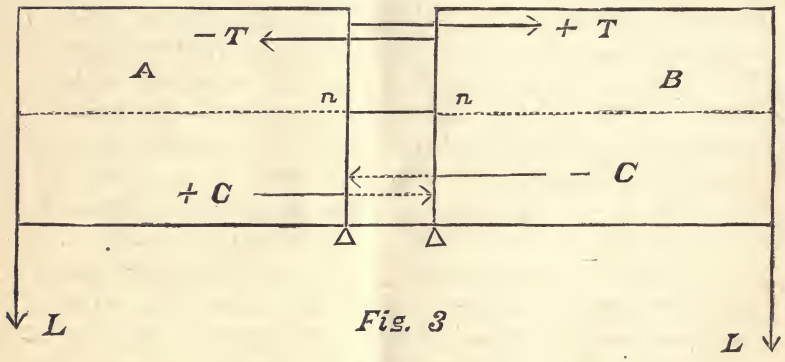

Fig. 3

of balancing, but it is done in machines directly, without the fiction of giving the shearing forces leverage.

The development of the static forces (T T) must be attended by their equal shearing forces, (C C) in this theory, or they can possess no leverage resistance to the rotation of the loads (L L). The identity of the pair of (T T) forces and the pair of (C C) in this equilibrium follows from the "parallelogran of forces"; for produce the lines of action of each of the T forces, backward, until each intersects that of its load L. From these points draw the two diagonals to the points of support and there decompose them and we will obtain the pair of T forces which now become a pair C forces. This also, demonstrates that a neutral line and

an uniformly varying compressive force cannot exist in this condition of equilibruim.

Resisting Moment.—A writer states that at any section, "The moment of the external force or forces will be equal but opposite in sign, to the internal resisting moment." A *resistance* pure and simple like that offered by a rope to extension or a column to compression can have no resisting moment, nothing but a force can have a moment. The internal resistance offered by the material of the beam is the passive measure of the intensity of the forces. The vertical load generates the horizontal forces through its ability to rotate the beam, the material passively *resists* this rotation and measures the intensity of the horizontal effort.

No error would result in calling it a "resisting moment" as a mark of identification, but when it is further said that "the sign of this moment must be opposite to that of the load" that produced it, a fatal error is introduced from an entire misconception of the subject; for it is conceded that the vertical applied forces generate the horizontal forces, each on its own side of the section of maximum stress.

It is a well recognized principle that like produces like and not unlike, that the energy of any movement or tendency is balanced by that which it generates in exhausting itself. Then to require the load on A, Fig. 1, to exhaust its rotating power by producing a right handed couple (T C) violates undisputed principles of nature.

If the load generated nothing but rotation and the balancing "couple" was of entirely different origin the statement would be true. From their symmetrical arrangement the horizontal forces, at the section of maximum stress, has the semblance of this truth, but at any other section it has not even this semblance.

Equilibrium at Any Section.—All writers on the common theory of flexure agree that at any section "the moment of the applied or deflecting forces equals the sum of the moments of the resisting forces.

Then let us consider the equilibrium, at the quarter sec-

tion of the half beam A, by removing the left hand quarter of the beam A B thus giving us Fig. 4. Between this quarter and the middle section the molecular movement of the material is to the left, above the neutral line, and to the right below this line. The direction of these movements demonstrate that there are only two forces acting between these sections, one *pulling* the material of the beam from and the other *pushing* it to the middle section. The forces *t* and *c* acting at the quarter section are then represented in direction by the arrows in Fig. 4, and there are no others.

In the "common theory" the load L, on B is balanced by the left-handed couple (T C) at the middle section, and all unbalanced vertical forces being thus provided for, we are lead to inquire from what outside source do the forces *t* and *c* come and what do they represent. They are, at this section the generated components of the load L, that rested upon the removed quarter of the beam A B and represent it in the system of forces given in Fig. 4 and the moment of the removed load, though forming apparently a left-handed couple must have been balanced, in the whole beam by the left-handed couple (*t c*) in defiance of the contrary law of statics—they do balance, though in accordance with our rule for substituting "applied" for generated forces; in order to determine the mechanical moments of the generated forces *t* and *c* when resisting the moment of the removed load on A, the center of rotation being at the bottom of the section

In discussing this equilibrium the "common theory" writers represent a right-handed couple as acting at this sectiou, composed of forces directly opposite in direction to *t* and *c*, which is a fiction, as we have just shown that *t* and *c*, as represented in Fig. 4, are the *only* horizontal forces that can possibly act at this section and produce the "deformation" that is known to take place. The fiction that there exists at this section a "resisting moment opposite in sign" composed of the resistance of the material to longitudinal tension and compression, will not do, for this resist-

ance of the material simply measures the horizontal intensity of t and c, and being *passive*, from the very definition can have no moment or *tendency* to rotate A. One class of writers[3] represent a right-handed "strain couple" and another[4] a right-handed "force couple" as acting at this quarter section, when in fact it is left handed *apparently*.

The forces t and c cannot balance each other, and, therefore, cannot be generated equal at this section, although it is a fundamental principle of the common theory that they should.

We have seen that in order that c shall be balanced by t, the beam A must transfer t as a shearing force until it becomes opposite c on its own line of action; this it cannot do

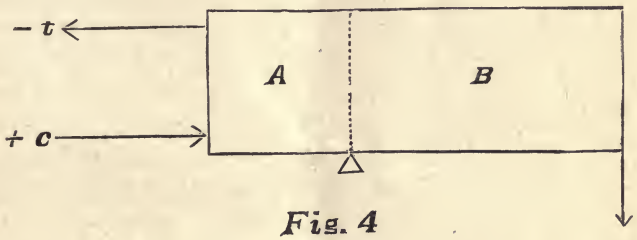

Fig. 4

without destroying the equality that is claimed to exist between the tension and compression at the middle section, for with a shearing force t acting opposite to c, at the quarter section, the resultant compression at the middle section will be $C—t$ as the compressive and shearing force act in opposite directions along the same line of action,—the resultant force $C—t$ will therefore be less than T.

Then as no shearing force can act in the opposite direction to that of c, at this or any other section, without diminishing the compression on the middle section, these *tension* and *compression* forces cannot balance at the middle or section of maximum stress. For these horizontal T and C forces

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(3) Weisbach's Mechanics of Eng., pp. 548 (Coxe).
(4) Wood's Elements of Analytical Mechanics, pp. 129.

increase from zero at the ends to their maximum value at the middle section of the beam. When the lever-arm of the load is zero the horizontal forces are zero, then each increment to the lever-arm of L, gives an increment to the value of T and C and if the forces cannot balance and develope equally at the section of their origin, the opportunity is lost and this condition cannot be imposed upon them after they come into existence, and this they cannot do, for it requires at *each* such section the horizontal effect of *at least* three forces to balance, which can only be fulfilled at the middle section.

Spring Experiments.—Certain experiments made by cutting the beam into two pieces and reconnecting the parts by placing elastic springs between them and opposite the resultants of the tension and compression of Fig. 3, thus giving it the semblance of the original, and the developement of equal tension and compression in the springs has caused some minds to conclude that the demonstration of the common theory "has all the absoluteness of geometry."

Prof. Rankine says of the right line balance of parallel forces when applied to a body, "that all pairs of directly opposed equal forces may be left out of consideration, for each such pair is independently balanced what soever its position may be." The horizontal forces in the whole beam consists of two such directly opposed pairs, and the body as a whole must remain at rest whether the forces of these pairs are equal or not; for there is no unbalanced force to cause horizontal motion. Now, if the whole is at rest without the forces of these paris being equal, must not the half of the whole be in the same condition? Then "the common theory" equality of the tension and compression in the beam, and its semblance, when its parts are connected by springs, cannot be a necessity from the first condition of equilibrium, but results from an entirely different mechanical principle.

Moment is the result of the arrest of rotary motion—when we pull a rope with a force at each end right line

motion is arrested but neither force possesses any leverage, but should these equal and opposite tensile forces be developed in arresting rotary motion, each would possess leverage in resisting the cause of the rotation in proportion to their distance from the center. This leverage it can only have from the bodys transmitting a shearing force to the center of rotation.

In the spring experiments the tension spring arrests the rotary motion and each half-beam transmits to the compression springs, the centre of rotation, equal and opposite shearing forces (in order to possess leverage) which causes the compression to develope equal to the tension, which is not a result of the first condition.

The half beams A and B, Fig. 3, connected by springs possess another property that is fatal to the supposed identity between it and the solid beam. After placing a pair of springs at the middle, divide the half beam into two parts at any section and reconnect them by means of another pair of springs on the same horizontal line with the first pair, then demonstration and experiment shows that the stress in each pair is the same, thus making the tension and compression uniform, from end to end, in the common theory beam, when in the beam as it is, they are zero at the ends and a maximum at some interior section.

This uniformity of stress, from end to end of the beam, in the "common theory," results from one of the oldest known principles of statics, "the parallelogram forces." In Fig. 1, prolong the lines of the forces L and T until they intersect, then the resultant would move A in the direction of its diagonal, which must be resisted by decomposing the resultant into its original components L and T, and apply C and S equal and directly opposite and equilibrium will be established. If we decompose the resultant of L and T at any point on the diagonal of A, we will obtain the original components; that is, the tension T does not vary in value. If we now make the same diagram of forces for the right hand half of the beam B, and place them together as

in Fig. 3, with a spring opposite to the line of action of the tensile forces and remove the applied T forces, then by applying the loads L to both A and B, the upper ends of the diagonals, being now connected, they will generate an uniform tensile stress between these points. And there can be no equilibrium of moments between the deflecting and resisting forces at any point in the beam, except at the section of maximum stress.

This failure of the stresses in "the common theory" beam to *decrease* longitudinally from the section of maximum stress, results from another "well recognized" principle of statics. An applied force distributes itself through the body on its line of action without increase or decrease, and its point of application may be taken anywhere on this line in the body. The applied forces T and C, Fig. 1, must follow this law and be of uniform tensity from end to end of A.

Here again it is seen that the common theory adopts a process that leads to results that are entirely the reverse of what is true for the beam.

Center of *Rotation*. From moments measuring the tendency of forces to produce rotation about a point, there must be, in every body that is acted upon by a system of forces that balance through the equality of their moments, a common center of such tendency to rotation, which determines the lengths of the lever-arms of the forces. And if the equilibrium of this system is thoroughly understood, the amount and character of the work performed by each individual force must be clearly ascertained—to know the tool that it works with, its lever-arm or distance from the center of rotation must be accurately determined. If the line of direction of a force passes through the center of rotation, the lenth of its lever-arm is zero, then by giving it a moment about some other point, we give it what it does not possess. This is the condition in the equilibrium of the common theory, the horizontal forces develop equal, from the pressure producing forces passing through the center of

rotation. When this is the case "we may take our origin of moments anywhere," but the results obtained express a mathematical play instead of a mechanical condition.

An algbraic equation may express the equilibrium as an arithmetical problem, but entirely misrepresent the mechanical condition. Had the common theory writers determined the actual center of rotation and formed their equation to express the mechanical conditions about this center, instead of as the R. R. Gazette* says as an apology for not doing so, the arithmetical results for the couple being "constant wherever be the origin of moments," which "shows that it is not necessary to assign the origin of moments," it would have condemned and shown the absurdities of the theory. To obtain this constant arithmetical result, when the origin is between the lines of action of the forces of the couple, their moments, being of the same sign, add; but when the center is outside, the moments of the forces must subtract, as they then have opposite signs, in order to maintain this "constant" arithmetical result. It does appear very important that we determine by locating the center of rotation, whether the pair of forces work together or work against each other.

The center of rotation has been variously located by different writers. Wiesbach says it is on the neutral line in the section of maximum stress. Another writer,[5] after locating it at the center of A figure 1, says it may be "anywhere;" for the couples (L, S) and (T C) "tend to turn the half beam around its center of *mass*. Equilibrium being established, the *tendency* to turn about any point within or without the beam is zero, and the origin of moment may be anywhere." The motion itself may be zero, but the *tendency* can never be zero while the forces possess leverage. Therefore our writer must let it remain at the center of *mass* of A, as it cannot be removed by bad logic from where worse mechanics located it.

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(5) R. R. Gazette, September 6th, 1889.

Another author[6] says : "The usual method of failure of beams is by cross-breaking, this is caused by the external forces producing rotation around some point in the section of failure"; but fails to locate this very important point.

In a *purely static* system of forces, such as the "common theory," the center of rotation is easily ascertained, for the shearing of each force travels directly to this point, and the intersection of their lines of travel locates it unerringly.

In the "spring experiments," and all other devices by which it is demonstrated that the tension and compression develop equal, the compressed spring, or whatever represents it, is, and must be, for this equality, the center of rotation or *tendency*. Then in the solid beam, the beam must be *able to rotate* around a point on the compressive force, which it cannot and has no appearance of doing during bending, therefore the compression and tension cannot be generated equal in the solid beam.

But in the beam as it is, we have only a vertical shearing force, which alone is not sufficient to locate the center, and we must combine with this the knowledge derived from the character of the "deformation" of the beam during bending. The vertical shearing forces locates the center of rotation in the section of maximum stress, and as this is also the only plane that remains "fixed," relatively, to the whole, during the process of bending, the center of rotation must be a point of this plane as it is a "fixed point" also.

If we divide a beam, that is supported at its middle and loaded at its ends, into very thin horizontal layers, each distinguished from the other by differences in color, then during the process of bending the points of each layer will be observed to have a downward motion, and that the points of no layer can rotate around a point, as a center, in a plane higher than its own—the points of the bottom layer can rotate around a point in its own plane, but not around

(6) Merriman's Mechanics of Materials, pp. 42.

a point in a plane higher than its own ; then, as there can only be one such point for the whole beam, it must be at the intersection of the plane of maximum stress with the plane of the concave or compressed side of the beam, where Galileo determined it to be nearly two and a half centuries ago.

Development of Equilibrium.—When an unbalanced force, such as a vertical load, is applid to a beam or body, and equilibrium is established either by the development or the application of forces acting at right angles to the direction of the applied, unbalanced force, two cases may be distinguished :

1. The resultant moment is the moment of a single force in the direction of each axis, or the compressive forces have no moment, as their lines of action pass through the center of rotation or *tendency.*

2. The resultant moment in the direction of the axis of the applied force only, is the moment of a single force, and in the direction of the other it is the sum of the moments of two components, or the compressive component has leverage as well as the tension, and neither have any shearing force. These distinctions the writer has never seen made by any text book of mechanics. When motion could take place in the body, or half body, as a whole, were it not for the conditions of "fixedness" imposed by connections outside of the body itself, then the arrest of the rotary motion of the body, as a whole, produces tension in the "fixing" farthest from the center of rotation and equilibrium of moments is established from both the applied force and the developed tension transmitting their shearing forces to the center of rotation.

In Fig. 1 we have the load L, the *reaction* of the support S, and equal and opposed to S the shearing force L, thus giving us the vertical effect of three forecs, the resultant moment of which is that of the single force L, for as both the opposed forces S and shearing L pass through the cen-

ter of rotation, their lever-arm is zero, consequently the moment of the three is that of L. An analysis of the equilibrium of the forces acting in the direction of the other axis or the (T C) forces leads to the same conclusion, for if they balance they do not dc so in any mysterious mythical way This principle is the basis of the equilibrium in the spring experiments, nut-crackers, the "common theory" and a host of other contrivances with which we are familiar.

But when the condition of "fixednes" is such as exists between the two halves of a beam, and each half is solicited to rotate in opposite directions around a fixed center, no motion can take place except through the "deformation" of the beam itself. This requires that its rigedness shall first be overcome by longitudinal compression in order that the half-beam may assume the arc form *first* by the shorting of the material on the concave side, the result of which is bending and the develoyment of tension. The pressure components are first generated and are not the shearings of the tensile forces,—they do not pass through the center of rotation and therefore possess leverage.

With eqnilibrium established for any given load L, under our first case, the order of development is as follows: An increment to the load L gives an increment to the rotation, its conversion into a tendency gives an increment to the tension, this in turn gives an increment to its shearing and this an increment to the moment of the tensile force by which equilibrium is re-established.

In the second case an increment to the load L gives an increment to the compression which gives an increment to the shortening, this admits of an increment to the bending (which is actual rotation and not a *tendency*), and this finally gives an increment to the tension, and equilibrium is re-established by the increments to the horizontal forces giving an increment to the sum of their moments without the increments balancing as right line forces.

THE WRITER'S THEORY.

The method of generating the horizontal forces, distinguished in our second case, has been made the basis of the theory given in my book on "The Strength of Beams and Columns"[7] The Railroad Gazettee of July 26, 1889, in its review of this book says: "If a theory containing a mechanical absurdity leads to rational results, it does not prove that the "fallacy" is truth, but rather that some other error must be "involved to offset the first error." The Gazette then proceeds to demonstrate the existence of the fallacy from glittering generalties and "mathematical necessities" drawn from the formulated principles of *pure* statics that are applicable, solely and alone, to the first case above given, with equal propriety, its fallacy might have been demonstrated from its failure to comply with "the well recognized principles" of Dynamics, Hydrostatics or those of any other kindred science. This method is certainly not within the scope of fair criticism.

The horizontal forces do not exist of themselves, neither do we apply them; they are generated. An unloaded beam has none, and if the load be applied pound by pound, the horizontal forces grow and strengthen from zero, with the increase in the load, and the method of their generation is the vital part of any theory of the *flexure* of beams and columns.

If the method of generation, described in my theory, is impossible mechanics, it can be demonstrated, *per se*, and not because there is another rational method that is applicable to entirely different mechanical conditions, and the conclusions drawn from the results of one process are at variance with those from the other.

No "common theory" writer, with whom I am familiar, makes the process of generating the horizontal forces any part of his demonstration. The conclusions are "mathe-

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(7) E. & F. N. Spon, publishers, New York and London.

matical necessities," drawn from general algebraic equations expressing possible equilibrium about every point in the universe, and the physical condition of the beam is only considered when the amount of the stresses are needed to give value to the forces of the general equation.

The Gazette again says. "Many results found by the author agree fairly with those of direct experiment, and in some cases it may give better results than those found by the common theory." Exact identity between the computed strength, from my theory, and the determined experimental strength could not be expected from the record of experiments already made, which was the only source I had from which to draw my illustrations. The *tensile* and *compressive* strength, as recorded, is usually the mean of many tests, the single test varying in value from five to ten per cent. from the mean values, while the computed transeverse strength is for a single test, which test strength may vary from the strength computed from mean values five to ten per cent. Had the number of tests been the same to determine the tensile, compressive and transverse strength then the computed strength, from the mean of the tensile and compressive tests, would be identical with the mean of the transverse tests. In all cases where the strength of the beam was computed from the tested *tensile* and *compressive* strength of the identical material contained in the beam broken, the discrepency was three per cent. and less, this is especially noticeable in the very careful experiments of Mr. Kirkaldy and those made at the Watertown Arsenal.

Columns.—The amount of the load applied to a column directly measures the amount of the internal stress, and their relation is the crucial test of the correctness of any theory of flexure. The "common theory" has utterly failed to establish any such relation,—Gordon's, the most successful formula, is based upon a direct denial of the correctness of the theory; in its demonstration, it is said, [8]

(8) Burr's Materials of Engineering, pp. 430.

'The condition of stress in any normal section of a long column is that of a uniformly varying system composed of a uniform stress and a stress couple,'' the *resultant* compression of this ''system'' is then greater than the *resultant* tension, which moves the *neutral line* from the center of gravity of the section. The writers's theory fully establishes this relation, the correctness of which is amply sustained by all recorded experiments.

R. H. COUSINS.

Austin, Texas, August 29, 1891.

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